Multi-body modelling of timing belt dynamics

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Abstract: Although timing belt drives have recently been increasingly used in mechanical design, their behaviour is still considered to a large extent to be unpredictable, especially under varying operative conditions. The acoustic emission of the transmissions, above all, has been thoroughly investigated in past years, but noise still represents an unresolved problem for many applications and a concern for belt designers; therefore, the availability of good predictive models would be very useful for both design and application phases. The present work describes a multi-body numerical model that has been developed for the characterization of the dynamic behaviour of timing belt transmissions, with the final goal of assessing the acoustic radiation of a given design. Modelling and simulation have been performed by means of commercial software packages, but more additional programming was required to obtain dynamic models capable of simulating the complex behaviour of toothed belt transmissions. Several experimental tests have been performed to identify the many parameters that influence system dynamics and to validate the resulting computer aided engineering (CAE) model.

Keywords: timing belts, lumped-parameter dynamic models, mechanical vibrations, acoustic radiation, multi-body simulation tools, automotive applications, virtual prototyping

NOTATION

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>boundary element method</td>
</tr>
<tr>
<td>CAE</td>
<td>computer aided engineering</td>
</tr>
<tr>
<td>fn</td>
<td>frequency of transverse vibration (Hz)</td>
</tr>
<tr>
<td>FEM</td>
<td>finite element method</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>h</td>
<td>tooth thickness on pitch circle (mm)</td>
</tr>
<tr>
<td>l</td>
<td>length of the belt span (m)</td>
</tr>
<tr>
<td>m</td>
<td>gear module (mm)</td>
</tr>
<tr>
<td>MSS</td>
<td>mechanical systems simulator</td>
</tr>
<tr>
<td>n</td>
<td>sprocket angular velocity (r/min), order of vibration mode</td>
</tr>
<tr>
<td>SLDV</td>
<td>scanning laser doppler vibrometer</td>
</tr>
<tr>
<td>T</td>
<td>belt tension (N)</td>
</tr>
<tr>
<td>vi</td>
<td>impact velocity (mm/s)</td>
</tr>
<tr>
<td>z</td>
<td>number of teeth</td>
</tr>
<tr>
<td>ρ</td>
<td>belt linear density (kg/m)</td>
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1 INTRODUCTION

Nowadays, regulations in the automotive field are increasingly focused on user safety, and machine design has to comply with increasingly strict parameters in order to meet the related requirements. Moreover, companies, especially in the automotive sector, have to keep up with a shorter time-to-market in order to satisfy new customers and to win larger market shares. The advent of the new economy puts a further stress on such trends and highlights the need for virtual prototyping tools able to grant shorter development times and assess beforehand innovative solutions. In the field of power transmission for automotive applications, for instance, much effort has been expended in past years on studying the noise characteristics of the transmission itself, but in many cases the acoustic emission still represents an unresolved problem and a concern for the designer. This is emphasized even more by the fact that a great deal of effort has recently been put into reducing noise from engines, tyres, covers, etc., in order to optimize vehicle cabin acoustics, and therefore noise from other sources becomes more remarkable and annoying to car passengers.

Within this context, the availability of good predictive models of the acoustic emission of timing belt transmissions would be very useful for both design and application phases. The present article describes a detailed numerical model that has been developed for the characterization of the dynamic
behaviour of timing belt transmissions, with the final goal of forecasting the acoustic radiation of a given design and suggesting possible design changes. In research previously developed at the University of Ancona [1], synchronous belt noise has been analysed by processing vibration data, and the method has been validated by comparing the results with experimental values obtained from acoustic intensity measurements. The work described here represents a further step of the research that outlines a fully numerical procedure where the vibration data analysed through the acoustic model are obtained from belt transmission design by means of a multi-body model of the mechanical equipment.

1.1 Literature survey

Synchronous belts are very common for transmitting power and motion in mechanical systems, and they are often preferred to more expensive and complicated drive components such as chains, gears or linkages. The flexibility of belts allows them to absorb shock loads, but on the other hand the related compliance plays an important role with respect to large amplitude vibrations and noise problems [2–5], which represent a key concern, especially for applications in the automotive industry. Unfortunately, the analysis of belt noise is a rather difficult task, and its correlation with belt mechanical vibrations is an even more complicated problem: in fact, the running noise generated in a power transmission belt can appear in a wide variety of types and may be caused by several generating mechanisms, as shown by Kubo et al. [6] and commented upon in more detail below. In addition, things are made more difficult by the fact that not all the noise detected near a belt drive arises from the belt itself; for instance, in the case of a belt operating in the power transmission drive of an internal combustion engine, bearings, fans and other mechanical pieces of equipment generate interfering noise that may be taken for belt noise.

A great amount of literature is available on the general subject of mechanical vibrations, and many studies have been published in past years on the dynamics of flat belts or V-belts, also with the development of powerful non-linear models [7]; recently, interesting models have been proposed in the related field of chain drive dynamics [8]. Few investigations, by contrast, have been performed on the noise generated by timing belts, and only some of these have been based on dynamic modelling of the transmission system [9]. Among them, several models, in order to study longitudinal vibrations [10] or transversal vibrations [11, 12], consider the transmission line divided into belt spans, each one treated like a string, and apply to them the basic equations of the belts. Only a few authors use lumped-parameter models such as the one proposed here: Gerbert et al. [13] developed a model for quasi-static loading, Uchida et al. [14] studied the load distribution for different engine specifications, Karolev and Gold [15] assessed the effects of variable torques and validated experimentally their models, and Kagotani et al. [16] dealt with the specific case of helical timing belts. Recently, Johannesson and Distner [17] proposed a multi-body model quite similar to the one described here, only neglecting tooth mass and without using linear dampers but concentrating on dry friction history which greatly affects belt expected life. Since multi-body models can be very accurate even if relatively simple, this particular kind of approach was chosen for the present work [18].

The present authors claim that, to their knowledge, the model described in this paper is the first one of this kind to study the transversal vibrations of toothed belts by means of numerical analysis. It must be noted, too, that very few studies focus their attention on the relationship between noise and vibration, and even fewer experimental studies are available on the same subject [19]. This is probably due to the difficulty of measuring both noise and vibrations in the case of belt transmission lines: the vibration velocity, in fact, can hardly be measured at the required high number of points while the belt is running by using conventional transducers. The identification of belt noise, on the other hand, is made difficult by the presence of many disturbing noise sources existing in the engine; moreover, a high spatial resolution of the measurement technique is needed in order to collect useful and significant information. The related work [1] has characterized the vibroacoustic behaviour of synchronous belts by laser velocimetry and has been useful in the phase of tuning the present multi-body model. The noise information from both the experimental measurements and the dynamic model has been processed by a boundary element modelling (BEM) tool to compare the results and give a qualitative model of the performance of an actual design, as explained in the following sections.

1.2 Main causes of belt noise

Noise radiation of toothed belts is due to several different causes, among which are [6]:

(a) impact generated by collision of the belt tooth against the bottom land of the sprocket at the beginning of meshing;
(b) impact generated by collision of the sprocket tooth tip against the bottom land of the timing belt at the beginning of engagement;
(c) collision between the flanks of the two teeth at the beginning of meshing;
(d) transverse and torsional vibrations of the belt;
(e) vibrations of the pulleys;
(f) airflow between belt and pulley;
(g) frictional noise coming from contact between fabric and steel.

Among these causes, two kinds of sound are far more powerful than the others [20]: the impact generated by the engagement between sprocket tooth tip and belt bottom land and the transverse vibrations generated by the belt when resonant conditions are induced by meshing frequency.
The impact velocity, \( v_i \) (mm/s), can be estimated by the following approximate expression

\[
v_i = \frac{\pi}{30} \left( \pi - \frac{h}{m} \right) mn - \frac{\pi}{180} \left( \pi - \frac{h}{m} \right) \frac{mn}{\rho} + \cdots \tag{1}
\]

where \( n \) is the sprocket angular velocity (r/min), \( m \) is the module (mm), \( z \) is the number of teeth and \( h \) is the estimated tooth thickness (mm) on the pitch circle (approximately the length of the tooth tip land of the wheel). Equation (1) shows that the impact velocity increases with belt speed and the polygon effect of the wheel. It is seldom affected by the number of teeth and is determined mainly by the module. In the case of usual toothed belts, the impact velocity is also affected by transverse vibrations which, in turn, depend on the flexural rigidity and tension of the belt itself.

As for the sound generated by transverse vibration of the belt, it must be pointed out that it is a forced vibration due to the polygon effect of the wheel. Therefore, when the wheel approaches an angular velocity yielding a meshing frequency that coincides with the natural frequency of the transverse vibration of the belt, such vibration becomes very large; Kubo et al. [21] and then Tokoro et al. [22] showed that the transverse vibration of the belt becomes stronger along with increasing belt tension and decreasing bending stiffness. Therefore, if both belt tension and natural frequency are high, the noise of transverse vibration of the belt becomes very large and masks all other noises.

The present research limited the investigation only to the noise coming from the transverse vibrations of the belt span, and the dynamic model has been built accordingly. Within this limitation, all the most important causes of belt vibrations during operative conditions have been taken into consideration, and the possible effects of their variation on system behaviour have been duly assessed.

2 MODELLING AND SIMULATION

2.1 Multi-body model of the transmission line

A lumped-parameter model, based on a discretization of the belt, has been defined to emulate the dynamic behaviour of the transmission: each tooth is represented by a rigid body with suitable mass and inertia (Fig. 1) connected by spring–damper constraints to the adjacent teeth and to the pulley; the wheels and sprocket, by contrast, have been modelled by means of rigid bodies. In order to manage properly the meshing phase and the polygonal effect, the geometry of the teeth (of both belt and wheels) has been duly reproduced. On the other hand, to facilitate modelling and simulation, a plane model has been developed, and linear components have been chosen to model belt elasticity (linear springs) and internal dissipations (linear dampers), while Coulomb dampers have been chosen to model contact friction. Johansson and Distner [17, 23] showed that linear springs and dampers can be properly used to model the characteristic response of the rubber tooth in the load ranges of a car engine, since the major source of non-linearity is the varying contact geometry and not the rubber non-linearity itself.

The described model is therefore composed of the following rigid bodies (Fig. 2):

(a) a driving sprocket, connected to the crankshaft, responsible for motion generation;
(b) one or more driven pulleys, each one connected to a camshaft;
(c) mechanical or hydraulic tensioners, in order to provide the desired preload to the belt (idler pulleys can be used instead);
(d) belt teeth, which can be joined at will to build up the transmission line.

The axial reaction was modelled by a spring and a damper; it must be noted that the stiffness of this spring can be easily obtained, because it is mainly due to the properties of the cord which is generally made of glass fibre or steel thread and therefore can be easily measured experimentally. The bending behaviour, on the other hand, is caused by both the cord and the rubber and was modelled through rotational springs and dampers.

One of the most important geometric parameters of the model is the relative position of the revolute joint connecting two adjacent teeth, since the meshing impacts exciting the belt and producing the transversal oscillations actually take place on the fulcrum itself. Two contact forces acting on a belt tooth have been considered: on the flank (with the conjugate flank of a pulley tooth) and on the tip (with the bottom land of a toothed pulley). Moreover, the contact forces acting at the belt bottom land (with the sprocket tooth tip) and on the back side (with an idler pulley or a tensioner) have also been considered. Therefore, proper sets of constraints have been placed at these points to model the
belt compliant behaviour during the meshing (mainly radial and axial deformations); of course, such constraints had to be activated only during contact phases, by means of triggers that were turned on according to the periodic motion of the transmission. The forces acting on each of these points were evaluated \textit{a priori} through a finite element method (FEM) simulation of teeth meshing under the same load conditions as the belt under examination (Fig. 3); the intensity, the line of action and the point of impingement of the internal reaction force proved to be parameters of great importance.

The model can be driven by external kinematic or dynamic constraints which allow the user to specify either the rotational velocity or the torque applied to the crankshaft and to the camshaft(s); it is also possible to define perturbing components (up to the fifth harmonic) in order to reproduce realistic behaviour of the belt drive.

Some of the described parameters were known \textit{a priori} from manufacturer’s data, obtained by experimental tests or found from the reference literature. Other parameters were totally unknown, though they were undoubtedly needed to implement the model. In this case their values were (roughly) identified by running several test case simulations and by comparing the experimental results with the numerical data, as shown below.

As for the geometry of the tooth (pitch, height width, cord position, etc.), this was imported from computer aided design (CAD) data provided by the belt manufacturer, while the mass properties (e.g. mass, position of centre of mass, inertia, etc.) were found accordingly by using the capabilities of the computer aided tools mentioned. Of course, all the data that were needed completely to define the transmission line (e.g. the number of teeth of the belt, the dimensions and the positions of the pulleys and of the idlers) were specified through an input interface, and then global congruence was checked (e.g. on the length of the belt or on the feasibility of the path).
2.2 Simulation tools

The model of the complete belt transmission was implemented by means of a multi-body code; such kinds of tool are largely used in engineering to simulate the motion of complex mechanical systems, the constitutive parts of which can be seen as a set of rigid or flexible elements connected to each other by several types of constraint. ADAMS was chosen for this project because no commercial package already provided all the necessary features for the implementation of the described model, so ease of extensions and open code structure were considered a priority in the choice. In particular, the version of ADAMS that was used was not able to manage the contact between bodies, and therefore it was necessary to describe this kind of interaction using an external FORTRAN subroutine able to control the occurrence of an impact between belt and pulleys and, in this case, to reproduce both the normal reaction and the friction force that originated from the contact itself. The realization of this macrocode was made possible because ADAMS makes available, at every simulation step, the values of the position, velocity and acceleration of each part composing the model; therefore, since the geometry of the system was known, all the needed kinematic and dynamic data were obtained. Such a feature of the tool was essential in order to know, and therefore to control, the movements of the different parts of the model.

At the end of dynamic simulation it was possible to obtain detailed information on system behaviour and also to assess the values of some important data that, owing to the small size of the teeth or the impossibility of applying sensors, could not be observed experimentally; for instance, Fig. 4 plots the time history of the contact force between tooth and pulley during two complete revolutions of the driving sprocket.

The ultimate aim of the simulations was to assess the transversal vibrations of the belt during its constant rotation in order to use them as an input for SYNOISE, a program based on the BEM technique, able to give a preview of the acoustic power emitted by the belt. Once again, interface software was written to extract internal data that were not available to the user in the standard configuration of the simulation tool. In particular, simulation results are normally provided by ADAMS according to a Lagrangian approach, since the time history of selected (travelling) points of the belt is made available in the local reference frame. On the other hand, the assessment of acoustic radiation can only be performed on the basis of data arranged in a Eulerian fashion, since the vibration of selected points that are in fixed positions must be evaluated in the global reference frame (see Fig. 5). Of course, this step was crucial also to validate the model and therefore to compare simulation results with the vibration measurements carried out on the test rig set up in the laboratory, as shown below.

Since ADAMS libraries do not allow the user to access this kind of data, it was necessary to write an external FORTRAN subroutine: once defined, the selected output points (in fixed positions of the free span relative to the global frame) and the displacements, velocities and accelerations of the travelling teeth were stored for each selected position.

The two simulation tools were interfaced by means of a purposely developed code written in a LabView environment; this program processed the output file created by the multi-body analysis in two steps, so as to create a file in the format required by the BEM package:

1. In the first step, the program reads the output file generated by ADAMS and builds a three-dimensional matrix containing the velocity time history of all the teeth.
2. In the second step, the program displays the vibrations of the belt spans (at selected points) and performs the a fast Fourier transform (FFT) of the raw data. Once a frequency value for the analysis of acoustic radiation has been selected (suitable values have been obtained by experimental tests), the program writes two files (for both the real and imaginary part of the signal) in a format that can be read by SYSNOISE. Of course, this output data must be sorted in such a way that a correspondence between the target points in ADAMS and the nodes of the mesh in SYSNOISE is kept.

3 VALIDATION OF THE MODELS

3.1 Description of the experimental set-up

In order to validate the model of the belt drive, laboratory measurements were performed on the belt stand illustrated in Figs 6 and 7. The box-shaped structure housed an electric motor, a cylinder head of a 16-valve internal combustion engine, immersed in an oil bath, and a cooling system. The motor and cylinder head were mounted on two separate vertical steel beams fixed to the stand base. Tables 1 and 2 show some characteristics of belts and transmission lines that were taken into consideration in the experiments according to the different tested layouts. It has been shown by Tokoro et al. [24] that great dynamic tensile variation occurs in synchronization with rotations of the camshaft, pulley and belt (mainly caused by the valve drive and, to a minor extent, by pulley eccentricity and belt cord non-uniformity). Therefore, in order to reproduce as well as possible the torque oscillations caused by conventional automotive engines, a real engine head dipped in lubricating oil was used, driven at the desired rotational velocity by a conventional three-phase, four-pole induction motor with a speed variator. In this way, the torque variations of the camshaft pulley depended directly on the camshaft dynamics; moreover, the immersion in the oil bath also made it possible to reduce the noise emitted by the test rig.

To set the initial tension of the toothed belt, a tensioning idler was used and, in order to reach the predefined initial
tension $T_0$ (N), the well-known formula of vibrating cords was applied

$$f_n = \frac{n}{2l} \sqrt{\frac{T}{\rho}}$$

where $l$ is the length of the belt span (m), $\rho$ is the belt linear density (kg/m), $n$ is the order of the mode and $f_n$ is the frequency of the related transverse vibration (Hz).

All the tests were performed using idle pulleys, mounted on eccentrics, to preload the belt. This choice was made because spring-based tensioning systems would have complicated the model by introducing difficult-to-characterize compliance; moreover, this solution is the most common layout in modern industrial and automotive applications.

3.2 Experiment sound power estimation and transverse vibration measurement

Three different kinds of measurement were performed by means of the instrumentation sketched in Fig. 8 and summarized in Table 3:

(a) total sound power radiation, that is, all the noise generated by all moving parts (belt, pulleys, idlers, etc.);
(b) vibrations of each component of the rig (belt, pulleys, idlers, etc.);
(c) vibrations of the belt only.

Acoustic intensity measurements were performed by a different working group in the same Department of the University of Ancona [25], and some results are commented upon here for convenience. The same reference also provides details on the measurement techniques used for the identification of the models and an assessment of the related uncertainty.

Some noise measurements were performed using acoustic intensity techniques in order to collect sound power spectra under operating conditions and to compare them with the numerical predictions of the BEM model. The sound intensity was measured by a two-microphone probe at a number of points aligned on a grid built around the belt drive using fine threads. The grid consisted of five lateral surfaces of a rectangular parallelepiped, divided into square elements of 10 mm side length; such measurement surfaces, together with the steel plate lying beneath the belt drive, formed a close surface that made it possible to estimate the total sound power emitted by the belt drive as the surface integral of the acoustic intensity measurements (according to ISO 9614-1).

Useful information was extracted by the power spectra of the signal; a typical sound power spectrum is illustrated in Fig. 9, where the reference value used for the description of the acoustic power levels in dB was $10^{-12}$ W. Several peaks appeared in the frequency range of interest, resulting from the multiplicity of noise sources present in the test stand. Generally, these peaks coincided with the harmonics of the rotating frequency, with the meshing frequency and with the natural frequencies of the belt strands.

By varying some parameters of the considered transmission, it was shown that at meshing frequency the power level increases by 4–6 dB every time the speed doubles (in accordance with theory, since the sound power is directly proportional to the square of vibration velocity), and that the power radiation increases by more than 3 dB per 10 mm increase in belt width.

### Table 1 Main parameters of the belts used for the tests

<table>
<thead>
<tr>
<th>Profile type</th>
<th>Pitch (mm)</th>
<th>Tooth height (mm)</th>
<th>Tooth width (mm)</th>
<th>Belt width (mm)</th>
<th>Cord</th>
<th>Body and teeth</th>
<th>Tooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parabolic</td>
<td>9.525</td>
<td>3.5</td>
<td>3</td>
<td>25</td>
<td>Spirally wound glass-made</td>
<td>Chloroprene compound</td>
<td>Nylon facing fabric</td>
</tr>
</tbody>
</table>

### Table 2 Main parameters of the transmission lines for the tested layouts

<table>
<thead>
<tr>
<th>Number of belt teeth</th>
<th>Layout 1</th>
<th>Layout 2</th>
<th>Layout 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of camshaft pulley teeth</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>Number of crankshaft sprocket teeth</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Number of idle toothed pulleys</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of idlers</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 8 Measurement chain
A boundary element analysis involving a vibrating structure requires the definition, either numerical or experimental, of the vibration velocity values of the equipment under test. In the present case, transverse vibrations were measured using a scanning laser Doppler vibrometer (SLDV). Such an instrument allows a fast scanning of the test surfaces, so that a great number of points were measured over each belt span (every 4 mm along the axial direction and every 8 mm along the transversal direction).

Attention was paid to the interference in the determination of the normal component of the velocity. This disturbing effect is caused by belt movement in the axial direction and can be reduced by keeping the incident laser beam as much as possible orthogonal to the running belt. Therefore, the belt drive under test was arranged with vertical shaft axes, which made it possible to position the laser head far away from the belt and then to restrict the vibrometer scanning angle. On the other hand, as a result of an increase in the distance of the measurement surface from the laser head, there was a reduction in the optical energy of the backscattered laser beam appearing to the vibrometer optics; a fairly good compromise was found by positioning the laser head at a distance of about 2 m from the belt span.

As in the case of noise measurement, several peaks were present in the measured spectra, and they could be associated with harmonics of the fundamental frequency of rotation and, in particular, with the meshing frequency. The expected belt span resonance frequencies were found in the spectra. If each belt span is considered as a string clamped at the edges, such frequencies can be evaluated by simply reusing equation (2) for each span.

In general, all the detected natural frequencies were slightly lower than their theoretical values. This is probably due to the effect of the axial velocity which, as shown in reference [26], makes the natural frequency of the axially moving belt span lower than that of the stationary belt. In some cases only a few peaks were clearly evident in the spectra; it emerged that in this case the meshing frequency coincided with one nth mode of one belt span. This resonance phenomenon is expected to occur under precise combinations of the test conditions, and in this case the vibration velocity profile over the belt span at meshing frequency becomes a mode shape and the transverse vibration amplitudes are stronger than under other resonance conditions, with the consequent influence on noise radiation.

Finally, the results were processed through the program SYSNOISE. The vibration velocities measured on the belt spans were the input data necessary to predict the acoustic radiation. In fact, the boundary element code converts experimental data into velocity boundary conditions to be applied for the acoustic field computation. It is noted that boundary element codes are actually becoming a ‘standard’ tool in the field of acoustic prediction because of their noticeable potential to import both data and geometry from other codes and to solve both interior and exterior acoustic problems. In this way it was possible to assess the sound power radiation caused only by the transversal vibrations of the belt span, without direct interference of impact noise, friction, wheel vibrations and so on. The same kind of processing was then performed on the corresponding output data of dynamic simulation, and the resulting numerical sound power radiation was compared with the experimental one, thus minimizing the effect of any interfering noise of pulleys or rig vibration.

A total of 18 different test cases were treated by varying the type of transmission line (layouts 1, 2 and 3), the initial tension of the belt (150 and 350 N) and the angular velocity of the driving wheel (1000, 1500 and 2000 r/min). Such results were also used in identifying the dynamic model and for final validation of the model itself, as described below.

### Table 3 Measuring instruments

<table>
<thead>
<tr>
<th>Instrumentation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanning laser Doppler</td>
<td>Measurements of belt velocities</td>
</tr>
<tr>
<td>Vibrometer</td>
<td></td>
</tr>
<tr>
<td>Spectrum analyser</td>
<td>Check of signal quality</td>
</tr>
<tr>
<td>Acoustic probe</td>
<td>Measurement of sound power radiation</td>
</tr>
</tbody>
</table>

![Fig. 9 Sound power spectrum of layout 3 (1500 r/min, preload 350 N). ‘L’ and ‘A’ are the linear and weighted sum of emitted sound power respectively](image-url)
3.3 Comparison between model behaviour and experimental results

More than 100 simulations were run in order to identify the values of the unknown parameters influencing the acoustic radiation and, finally, to validate the multi-body model. The vibration velocities computed by the mechanical systems simulator (MSS) software for each belt span were the input data necessary for the BEM code to predict the acoustic radiation; the results were then compared with the experimental data measured on the test rig.

Some parameters proved to be unimportant because a change in their values produced no significant variations in the emitted sound power. The radial damping, for instance, is among the less influential parameters, as shown in Table 4; in fact, the contact forces are mainly exchanged between the wheel tooth tip and the belt bottom land and, to a minor extent, at the contact interface of the flanks of the belt and wheel. These results support the model because the numerical behaviour is similar to the experimental behaviour. However, when a parameter did not influence the acoustic radiation, it was deleted from the model.

It was decided to analyse the vibroacoustic behaviour of the belt only at meshing frequency and under resonant conditions for two basic reasons:

1. It was shown in a previous study [1] that there is a strong relation between the vibration of the belt spans at meshing frequency and the total acoustic power emitted at the same frequency.

2. The test stand was suitably designed for the purpose of introducing disturbances in the form of camshaft pulley rotational vibrations or eccentricities, which can directly [11] or parametrically [3] give rise to resonance phenomena.

Table 5 summarizes some results for the second and third layouts. With a decrease in the bending stiffness, the noise decreases accordingly until a lower limit is reached and it then remains constant. This behaviour of the model, confirmed by experimental data, can be explained very simply. One of the highest sound power radiations occurs at the meshing frequency. Therefore, the variation in a parameter moves the resonance of the belt span away from the meshing frequency, the contribution of this parameter to the total sound power decreases over and over, and the total sound power of the belt remains more or less constant. This phenomenon is also evident in the histogram of Fig. 9: the sound power radiation has one of the highest peaks at meshing frequency, in this case at 540 Hz. Thus, the weight of the parameter in the model can be somehow considered as the capability of changing the resonance frequency of the belt.

Table 6 shows that the acoustic radiation forecast by the computer aided tool is usually some 13 dB higher than the experimental data, which is a significant difference. One factor potentially influencing the final result is the accuracy of the measurements. In order to verify the

### Table 4 Sensitivity of the model to changes in the value of radial damping

<table>
<thead>
<tr>
<th>Preload (N)</th>
<th>Speed (r/min)</th>
<th>Bending stiffness (N/mm/deg)</th>
<th>Bending damping (N/mm/deg/s)</th>
<th>Radial damping (N/mm/s)</th>
<th>Back damping (N/mm/s)</th>
<th>Point of applied force (mm)</th>
<th>Angle of applied force (rad)</th>
<th>Sound power (dB)</th>
<th>Experimental sound power (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layout 3</td>
<td>350</td>
<td>2000</td>
<td>17.3</td>
<td>0.1</td>
<td>1.0</td>
<td>0.49</td>
<td>1.44</td>
<td>0.175</td>
<td>78.8</td>
</tr>
<tr>
<td>Layout 3</td>
<td>350</td>
<td>2000</td>
<td>17.3</td>
<td>0.1</td>
<td>1.2</td>
<td>0.59</td>
<td>1.44</td>
<td>0.175</td>
<td>79.0</td>
</tr>
<tr>
<td>Layout 3</td>
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<td>17.3</td>
<td>0.1</td>
<td>0.0</td>
<td>0.59</td>
<td>1.44</td>
<td>0.175</td>
<td>78.9</td>
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### Table 5 Sensitivity of the model to changes in the value of some significant parameters

<table>
<thead>
<tr>
<th>Preload (N)</th>
<th>Speed (r/min)</th>
<th>Bending stiffness (N/mm/deg)</th>
<th>Bending damping (N/mm/deg/s)</th>
<th>Radial damping (N/mm/s)</th>
<th>Back damping (N/mm/s)</th>
<th>Point of applied force (mm)</th>
<th>Angle of applied force (rad)</th>
<th>Sound power (dB)</th>
<th>Experimental sound power (dB)</th>
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</table>

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sensitivity of the BEM model to the accuracy of velocity measurements, an uncertainty analysis of the computational values was conducted [25]. In fact, there were at least two interfering inputs during the laser velocity measurements, namely the generation of speckle noise (introducing a random error) and, owing to the non-orthogonality of the measurement direction with respect to the surface, the addition of part of the tangential component to the desired normal component in the results (generating a systematic error). As an outcome of this analysis, it has been pointed out that, for an uncertainty of 20 per cent in the velocity measurements (which is a reasonable value for the present application), the maximum deviation in the acoustic power is around 2 dB; a similar value could be expected for microphone-based acoustic measurements.

Another possible cause of the bias between the predicted acoustic emission and the measured values comes from the discretization of the continuous mechanical system in the lumped-parameter dynamic model. In fact, numerical models usually appear ‘stiffer’ than physical models, as explained, for instance, in references [27] and [28]. This effect is even more evident in the case of non-linear behaviour of the real system, which is usually linearized in the lumped-parameter model. This is actually the present case, where the rubber material constituting the belt has a highly non-linear behaviour; moreover, while meshing, the belt undergoes a relaxation process that progressively decreases the tension and therefore the stiffness [29].

One further issue potentially affecting the results of this particular analysis must be considered. The experimental data made it possible to build up a spatial model since the belt width was scanned in order to identify torsional vibrations. The simulation, on the other hand, was based on a plane model so that, in order to match the results, the numerical outputs of the two-dimensional multi-body model were distributed along the belt width. In this way, all the points of the belt spans were vibrating in coincident phase, which does not happen in reality, because of the presence of torsional oscillations that dephased the vibrations. Figures 10 and 11 show the different shapes assumed by the belt in the three-dimensional experimental model and in the plane multi-body one.

Figure 10, for instance, shows the output of the SYSNOISE software for the third layout, with a 350 N preload at 2000 r/min, allowing an easy comparison between numerical and experimental results: the belt shape shows the displacements when excited at the first resonance frequency, while the displayed grey levels relate to the transversal velocity. In this case the expected (numerical) sound power radiation was 77.0 dB, the same value actually measured on the test stand. Figure 11 shows the same kind of data for the second layout, with 150 N preload at 1500 r/min: the expected numerical sound power radiation was 65.8 dB, while a value of 50.4 dB was actually measured in this case.
Finally, several tests showed that the model is somehow coherent because, when the layout is changed but the same values are kept for the parameters, the gaps between the numerical and the experimental data remain similar (see Table 6). In the authors’ opinion, the bias of about 12–13 dB shown by this complex model is acceptable, and the results can still be considered significant, especially taking into consideration the coherent behaviour shown when the transmission layout is changed.

Fig. 10  Sample output of the SYSNOISE software (layout 3, 2000 r/min, 350 N preload)

Fig. 11  Sample output of the SYSNOISE software (layout 2, 1500 r/min, 150 N preload)
4 CONCLUDING COMMENTS

This paper has described a detailed numerical model that has been developed for the characterization of the dynamic behaviour of timing belt transmissions, with the final goal of forecasting the acoustic radiation of a given design and suggesting possible design changes. Modelling and simulation of toothed belts have been performed by means of commercial software packages, but additional programming was required to tune the models and to interface the tools. Several experimental tests have been performed for the identification of the most important parameters, and eventually a correlation has been found between experimental data and the CAE model.

To this end, several relations describing the behaviour of the belt tooth and, above all, the belt/pulley contact have been worked out. Some design parameters were rather easy to identify through a literature survey or experimental measurements, but other dynamic properties proved to be much more difficult to obtain, even by means of experimental tests. Therefore, some parameters were determined by means of a sensitivity analysis comparing the vibration velocities resulting from simulations with experimental data. Only the key parameters (e.g. axial belt stiffness, belt preload and velocity, etc.) were eventually retained in the model. It has been shown that belt vibrations and emitted acoustic power follow the same trends when varying system parameters, even if the simulated acoustic radiation is still some 12 dB higher than actual radiation.

In the authors’ opinion, such a difference between numerical results and experimental values may be at least partly due to the following effects:

1. The lumped-parameter numerical model discretizes the continuous mechanical system in a finite number of rigid bodies and (linear) springs and dampers, so frequency and amplitude of vibrations are affected.

2. The numerical model, which is a plane model, can be considered to consist of masses vibrating in-phase relative to the motion plane; this causes constructive interferences giving rise to maximum sound power emission. The experimental model, by contrast, is a spatial model and is therefore influenced by several mode shapes, so not all the points are in-phase and not all the interferences are constructive.

3. The experimental measurements were affected by noise, which could have caused a signal reduction and eventually a lower measured sound power.

In conclusion, the complex multi-body model that has been described is able to simulate very well the kinematics and dynamics of timing belt transmissions and provides a rough estimate of sound power emission. Nevertheless, when the most important belt parameters are changed, the trend of variation in acoustic radiation is well reproduced, and therefore quantitative estimates of the final result can also be provided. This tool is presently being used as CAD support to help evaluate beforehand the effect of design choices, and it is currently being inserted in the firm’s virtual prototyping chain.

REFERENCES


